

# Corrections on "A Gentle Introduction to Empirical Process Theory and Applications" by Bodhisattva Sen

1. Page 16, Exercise (HW1), the hat sign in  $\{\hat{f}_n\}$  can be removed to save some notations.
2. Page 19, Equation (7), the summation should be  $\sum_{k=1}^N$  (instead of  $\sum_{i=1}^N$ ).
3. Page 20, Exercise (HW1), it should be the entropy  $\log N(\epsilon, \mathcal{F}_L, \|\cdot\|_\infty)$  (instead of the covering number  $N(\epsilon, \mathcal{F}_L, \|\cdot\|_\infty)$ ) in both occurrences.
4. Page 21, Exercise (HW1), the function classes  $\mathcal{F} + \mathcal{G}$  and  $\mathcal{F} \cdot \mathcal{G}$  should be specified as summations and products of functions from  $\mathcal{F}$  and  $\mathcal{G}$ , respectively. The current notation is a bit misleading.
5. Page 33, Lemma 3.19 Hint,  $\tilde{\theta}_n := \alpha \hat{\theta}_n + (1 - \alpha)\theta_0$ .
6. Page 36, line 2, maybe need a  $K$  factor in the first term, e.g.,

$$\sqrt{\log N(\epsilon, \mathcal{G}_n(R), L_1(Q_n))} \frac{K}{\sqrt{n}} \sup_{g \in \tilde{\mathcal{G}}} \|g - g_0\|_n + K\epsilon.$$

7. Page 36, Exercise (HW1), the distance/norm  $Q$  in the bracket number is not defined. Is it should be  $L_1(Q_n)$ ?
8. Page 38, Theorem 3.24, using the argument in the proof, one can only obtain

$$\mathbb{P}(|Z - \mathbb{E}(Z)| > t) \leq 2e^{-t^2/2 \sum_{i=1}^n c_i^2}.$$

To have a tighter result, we have to bound the difference  $\Delta_i(x_1, \dots, x_{i-1}, x)$  more carefully, e.g., show that  $B_i - A_i \leq c_i$ , where  $A_i < \Delta_i(x_1, \dots, x_{i-1}, x) < B_i$ . In the current proof  $B_i - A_i$  is upper bounded by  $2c_i$ .

9. Page 40, some arguments are missing between bounding the difference term  $g(x_1, \dots, x_n) - g(x_1, \dots, x_{i-1}, x'_i, x_{i+1}, \dots, x_n)$  and applying Theorem 3.24. Note that  $g$  is defined based on  $f$  and  $Z$  is defined as the supremum over  $f \in \mathcal{F}$ .

10. Page 41, line 3, the constant in the upper bound (which is 4) does not match that in Lemma 3.16 (which is  $2\sqrt{6}$ ).
11. Page 79, Remark 7.1 first line, change VC indices to VC dimensions.
12. Page 98, proof of Proposition 8.2, first equation, would it be better to write  $\mathbb{P}(Z \geq t) = \inf_{\lambda > 0} \mathbb{P}(e^{\lambda Z} \geq e^{\lambda t}) \leq \dots$ ?
13. Page 98, proof of Proposition 8.2, it says “it can be checked that for  $z > -1\dots$ ”. However the infimum is attained at  $\lambda = \log(1 + z)$ . As  $\lambda > 0$ , we only need to consider  $z > 0$ , which also always holds as  $z$  is taken to be  $t/\nu$ .
14. Page 99, proof of Proposition 8.2, last paragraph of the proof, is it necessary to introduce the extra variable  $s$  in  $\gamma(s) := \sup_{\lambda \in [0,3)} (\lambda s - \psi(\lambda))$ ? Why not just use  $\gamma(t)$ ?
15. Page 102, Example 8.9, “let  $Z := n\|\mathbb{F}_n - F\|_\infty$ ”. Moreover, why  $\sigma^2 = 1/4$ , or is it just an assumption on  $\mathcal{F}$ ?
16. Page 103, Example 8.9, do we require  $c_3 > 1/2$  instead of  $c_3 > 0$ ?
17. Page 103, Example 8.9, some argument is missing for the scenario  $0 < x < 2c_1$ . Moreover, why we have an extra constant  $c_2$  in the final upper bound?
18. Page 155, Theorem 12.6,  $\mathbb{G}_n$  is not defined. We believe it is just the same as  $\mathbb{P}_n$ .
19. Page 155, proof of Theorem 12.6, in the introduction of  $\tilde{\mathbb{M}}_n(h)$  and  $\hat{h}$ , we believe it should be  $\mathbb{G}_n$  instead of  $\mathbb{G}$ .
20. Page 156, proof of Theorem 12.6, it should be  $P(f_{n,g}f_{n,h}) - P(f_{n,g})P(f_{n,h}) \rightarrow \dots = h^\top \mathbb{E}[\mathbb{G}(\dot{m}_{\theta_0}\dot{m}_{\theta_0}^\top)]h$ .
21. Page 156, proof of Theorem 12.6, last sentence,  $\tilde{\mathbb{M}}_n(h)$  converges weakly to  $h^\top \mathbb{G}_n(\dot{m}_{\theta_0}) + 1/2 h^\top V h \dots$ .
22. Page 157, first equation, it should be  $\tilde{\mathbb{M}}_n(h)$  instead of  $\tilde{M}_n(h)$ . Second line of same equation, missing a  $n^{2/3}$  in the second term.
23. Page 157, second equation, second line, should use  $\theta^*$  instead of  $\theta_0$  in all occurrence (also in the definition of constant  $b$  at the end of this example.)
24. Page 158, we believe  $\mathbb{E}(Z) = 0$  is assumed when calculating  $\text{Var}(Z)$ .

25. Page 166, the equations under definition of  $F(\lambda)$ , it should be  $\text{Ent}(g(X)^2) = \lambda \mathbb{E}(Ze^{\lambda Z}) - \mathbb{E}(e^{\lambda Z}) \log \mathbb{E}(e^{\lambda Z}) = \dots$