PhD Projects with Jelena Grbić

My research interests lie, broadly saying, in the area of Homotopy Theory which is a discipline of Algebraic Topology. More precisely, I am studying unstable homotopy theory and its applications to Toric Topology. Of recent interest to me is Cobordisam Theory and closely related to it stable homotopy groups of spheres. As it is to expect I would like to supervise PhD students who would be interested in one of these topics. To make my research world closer to you I will outline some possible problems in these areas.

Problem 1: Homotopy Exponents. Within Unstable Homotopy Theory I like to look at the homotopy exponents of topological spaces. To any topological space X we associate a sequence of abelian groups $\pi_n(X)$ which we call higher homotopy groups of X. They are homotopy invariants of X, and to calculate them tends to be a very difficult problem. Being abelian groups, we can look at their p-torsion parts and ask for the existence of p-exponent $\exp_p(X)$ for the whole sequence $Tor_p(\pi_n(X))$. Even for the simplest space of one cell, sphere, the whole information is unknown. If p is prime, then $\exp_p(S^{2n+1}) = p^n$; but if $p = 2$, it is known that a 2-exponent exists but not what it is. So as you can guess there is a lot of things to be investigated. For example, what are the homotopy exponents of Lie groups, homogeneous spaces, and so on.

Problem 2: Homotopy Decompositions of Loop Spaces. It is standard in mathematics to investigate an object by breaking it into smaller pieces, investigate the pieces individually, and then reassemble that information to gain insight into the original object. Therefore in classifying any mathematical structure, it is helpful to analyse the irreducible or indecomposable components first.

By considering loops in a space X, we are obtaining a larger space ΩX which can possibly be decomposed and which homotopy groups are closely related to those of X, namely, $\pi_n(\Omega X) \cong \pi_{n+1}(X)$. So by decomposing ΩX, we transfer the exponent problem from X to the exponent problem of irreducible factors of ΩX which might be easier to approach.

Within this problem, I propose to study a very important homotopy invariant of certain indecomposable factors, namely, the homology of the atomic retract containing the bottom cell of the loop suspension of a Hopf invariant one complex. Although this problem is interesting in its own right, its solution would provide an important link between algebraic topology and representation theory. It turns out that the problem of calculating the homology of the bottom piece of the loop suspension of an arbitrary complex is equivalent to the fundamental problem in the modular theory of the symmetric group, namely, determining possible decompositions of the identity. Mathematicians working in representation theory have tried hard to solve problems related to the fundamental problem of the representation theory of the symmetric groups in the modular case (when the characteristic of the field divides the order of the group) for over a hundred years, after the rational representation theory of symmetric groups was done by Young diagrams. In spite of much effort, the modular representation theory of the symmetric groups remains, for the most part, a mystery.

Problem 3: Toric Topology. I am also very much interested in Toric Topology, the study of topological spaces with an associated structure coming from an action of a torus. This area of Topology has various connections to combinatorics, homological algebra, invariant theory, group and Lie group theory, commutative algebra, modular representation theory and modules and algebras over the Steenrod algebra. I'll just emphasize that Toric Topology is a source of new and quite often unexpected connections between topology and algebra and differential symplectic geometry.

For a summary of the subject and possible projects I refer you to the Manchester Toric Topology Page at
`http://www.maths.manchester.ac.uk/ nige/tortop.html`

Problem 4: Geometrical realisation of elements of the stable homotopy groups of spheres. Coming back to the problem of describing the homotopy groups of spheres, we can study Cobordism Theory which in a purely geometrical way tries to construct elements of the stable homotopy groups of spheres $\pi_n^s(S^0)$. Cobordism Theory has a manifold as the main object of it study. Manifolds (generalisations of Euclidean space) can be enhance with additional structures such as vector fields, framing, boundaries (with corners), those coming from group actions, and so on. All these extra structures can help us studying manifolds and possibly creating new representatives of $\pi_n^s(S^0)$.

One of the problems I would like to investigate is wether we can obtain new elements of $\pi_n^s(S^0)$ coming from manifolds with corners.