ASPECTS OF HOMOTOPY THEORY
ABSTRACTS

Piotr Beben (University of Southampton)

Title: *LS-category and Moment-Angle Complexes*

Abstract: Given a simplicial complex K with n vertices, the moment-angle complex $\mathcal{Z}_K$ is a certain subcomplex of $(D^2)^{\times n}$ defined purely in terms of K. Despite their simple description, they have connections with several areas of mathematics, while at the same time their topology is very intricate. We will discuss the computation of their Lusternik-Schnirelmann category and homotopy type, and the various applications of this.

Alexander Berglund (Stockholm University)

Title: *Rational homotopy theory of automorphisms of manifolds*

Abstract: There is a classical programme for understanding diffeomorphisms of high dimensional manifolds whereby one studies, in turn, the monoid of homotopy automorphisms, the block diffeomorphism group, and finally the diffeomorphism group. The difference in each step is measured by, respectively, the surgery exact sequence and, in a range, Waldhausen's algebraic K-theory of spaces.

I will talk about joint work with Ib Madsen, in which we calculate the stable rational cohomology of the block diffeomorphism group of the g-fold connected sum $\#^g S^d \times S^d$ relative to a disk ($2d > 4$). Our result is expressed in terms of a certain decorated graph complex, which, quite surprisingly, is related to the "hairy graph complex", introduced recently by Conant-Kassabov-Vogtmann in the study of automorphism groups of free groups. I will also comment on the relation to the results of Galatius and Randal-Williams on the stable cohomology of the diffeomorphism group.

Natalia Castellana (Universitat Autònoma de Barcelona)

Title: *Cellular properties of fusion systems, Lie groups and p-compact groups*

Abstract: (with Ramon Flores and Alberto Gavira)

Let G be a finite group. In the homotopy theory of p-completed classifying spaces of finite groups (in the sens of Bousfield-Kan), the transfer map in stable homotopy theory provides an stable splitting describing BG as a retract of BS at a prime p, where S is a p-Sylow subgroup of G.

In unstable homotopy theory this is not true, but we could ask if BG can be built out of BS in the sense of cellular approximations described by Dror-Farjoun. Given connected pointed spaces A and X, X is A-cellular if it can be built from A by means of pointed homotopy colimits, possibly iterated. Many authors have contributed to the development of A-homotopy when the spaces involved are classifying spaces (see work of R. Flores, R. Flores-Foote, R- Flores-J. Scherer). Recently, importat results have been obtained by W. Chacholski, E. Dror-Farjoun, R. Flores and J. Scherer in describing cellular covers of nilpotent Postnikov stages.

In the more general case of classifying spaces of saturated fusion systems $(S, \mathcal{F})$ (defined by the p-completion of a centric linking system associated to it), with A. Gavira, we show that cellular properties with respect to finite p-groups are controlled by strongly $\mathcal{F}$-closed subgroups. In a joint work with R. Flores and A. Gavira we apply these techniques to the class of classifying spaces of compact Lie groups and p-compact groups in order to understand their cellular properties with respect to finite p-groups and Prufer groups.

Ralph Cohen (Stanford University)

Title: *Topological Field Theories, and how to compare them*

Abstract: I will describe joint work with Sheel Ganatra, in which we prove an equivalence between two chain complex valued topological field theories: the String Topology of a manifold M, and the Symplectic Field theory of its cotangent bundle, T^*M. This generalizes results of Viterbo, Abbondandolo and Schwarz, Abouzaid, and others. We use recent work of Kontsevich and Vlassopolous which describes two notions of duality among A_∞-algebras and categories over a field k, and show how they give rise to field theories. These are referred to as "Calabi-Yau" properties, and are related to the two dimensional cobordism hypothesis viewpoint of Lurie. We prove that the string topology category $\mathcal{S}(M)$ defined by Blumberg, myself, and Teleman is equivalent to the wrapped Fukaya category of the cotangent bundle, $\mathcal{W}(T^*M)$ as defined by Fukaya, Seidel and Smith, by an equivalence that preserves the Calabi-Yau condition. Finally we show how Koszul duality affects the Calabi-Yau condition, and how it implies a duality relationship between field theories.

John Greenlees (University of Sheffield)

Title: *Rational equivariant cohomology theories with toral support*

Abstract: For a compact Lie group G one may consider equivariant cohomology theories with values in rational vector spaces, and the talk is about those which only depend on isotropy which lies in some maximal torus. One can describe an algebraic model $A(G, toral)$ based on Borel cohomology for fixed points under toral subgroups and an Adams spectral sequence based on it always converges. Interesting ingredients include detection results, Solomons theorem from invariant theory and of course Borels basic results on the cohomology of classifying spaces. There are excellent prospects for a complete algebraic model in the form of a Quillen equivalence.

Ian Leary (University of Southampton)

Title: *Uncountably many groups of type FP*

Abstract: A group is type F if it has a finite classifying space. In particular, groups of type F are always finitely presented. Type FP is an 'algebraic shadow' of type F. In the 1990's Bestvina and Brady showed that not every group of type FP is finitely presented, solving a famous open question. I will talk about this and my new result which says that there are more FP groups than one might expect.

Brendan Owens (University of Glasgow)

Title: *Intersection lattices of 4-manifolds with boundary: some applications and problems*

Abstract: The intersection form on the second homology group is a fundamental invariant of 4-dimensional manifolds. In particular it was observed by Milnor, using work of Whitehead, that the intersection form classifies simply-connected 4-manifolds up to homotopy equivalence. In this talk we will discuss certain questions about 3-manifolds and knots which involve the intersection forms of smooth bounding 4-manifolds.

Nansen Petrosyan (University of Southampton)

Title: *Spin structures on almost-flat manifolds*

Abstract: Almost-flat manifolds were introduced by Gromov as a natural generalisation of flat manifolds. They occur naturally in the study of Riemannian manifolds with negative sectional curvature. It is well-known that every complete non-compact finite volume manifold with pinched negative sectional curvature is diffeomorphic to the interior of a compact manifold whose boundary is a finite disjoint union of almost-flat manifolds. Gromov and Ruh gave a topological description of almost-flat manifolds showing that every such manifold is infra-nil.

In this talk, I will use this description of almost-flat manifolds to discuss a necessary and sufficient condition for such manifolds with cyclic holonomy to admit a Spin structure. This condition is quite practical to check given a finite presentation of the fundamental group of the almost-flat manifold. In particular, I will discuss how it applies in dimension 4.
This is joint work with Anna Gasior and Andrzej Szczepanski.

Oscar Randal-Williams (University of Cambridge)

Title: *Infinite loop spaces and positive scalar curvature*

Abstract: It is well known that there are topological obstructions to a manifold M admitting a Riemannian metric of everywhere positive scalar curvature (psc): if M is Spin and admits a psc metric, then its Dirac operator must be invertible and so its $\hat{A}$ genus must vanish. In fact, if M is simply-connected as well as Spin, then deep work of Gromov–Lawson, Schoen–Yau, and Stolz implies that the vanishing of (a small refinement of) the $\hat{A}$ genus is a sufficient condition for admitting a psc metric.

I will discuss a related but somewhat different problem: if M does admit a psc metric, what is the topology of the space $\mathcal{R}^+(M)$ of all psc metrics on it? I will explain a recent result—which is joint work with Boris Botvinnik and Johannes Ebert—in this direction, and then focus on the homotopy theoretic questions that it raises.

Samson Saneblidze (A. Razmadze Mathematical Institute, Tibilisi)

Title: *The cup-one product and homotopy commutativity of loop spaces*

Abstract: We formulate the notion of a higher order derivation of a map with respect to a family of maps indexed by the cells of permutahedra P_n and apply it to the iterated $\smile_1$-products to construct a triple operation on the double loop space homology $H_*(\Omega^2 X)$ for a 2-connected topological space X. This operation is meant to be the first obstruction to the homotopy equivalence of $\Omega^2 X$ with a commutative topological monoid.

Svjetlana Terzić (University of Monetnegro)

Title: *The foundation of $(2n,k)$-manifolds*

Abstract: We introduce the class of toric $(2n,k)$-manifolds, which conisits of closed, smooth manifolds M^{2n} equipped with a smooth effective action of the compact torus T^k, $1 \leq k \leq n$ and an open T^k-equivariant map $\mu : M^{2n} \to \mathbb{R}^k$ whose image is a convex polytope, where $\mathbb{R}^k$ is considered with the trivial T^k-action. These manifolds we axiomatize by requiring the additional relations between the smooth structure of a manifold, the given torus action and an almost moment map μ.

The class of $(2n,k)$ manifolds contains many interesting and important manifolds such as quasitoric manifolds, complex Grassmann manifolds $G_{k+1,q}$ and full flag manifolds F_{k+1} equipped with the canonical action of the torus T^{k+1}, and the complex projective spaces $\mathbb{C}P^n$, $n = \binom{k+1}{q} - 1$ equipped with T^{k+1}-action obtained as the composition of the representation $T^{k+1} \to T^{n+1}$ given by the q-th symmetric power and the standard action of T^{n+1} on $\mathbb{C}P^n$.

We show that our axiomatization of $(2n,k)$ manifolds enables us to describe the topology of the orbit space M^{2n}/T^k in terms of naturally arising data from the given smooth structure and an almost moment map μ. Furthermore, under some additional assumption on these data we prove that the orbit space M^{2n}/T^k is homeomorphic to $S^{k-1} * F$, for some closed subspace $F \subset M^{2n}$.

The talk is based on the results obtained jointly with Victor M. Buchstaber.

Sarah Whitehouse (University of Sheffield)

Title: *Derived A-infinity algebras*

Abstract: The notion of a derived A-infinity algebra arose in the work of Sagave as a natural generalisation of the classical notion of A-infinity algebra, providing a framework for a minimal model theorem for differential graded algebras over a general commutative ground ring. I will discuss various aspects of these algebras, including joint work with Livernet and Roitzheim giving an operadic interpretation. Further developments of this work, joint with Aponte Roman, Livernet, Robertson and Ziegenhagen, include the study of representations of derived A-infinity algebras.

Jie Wu (National University of Singapore)

Title: *Combinatorial approaches to classical homotopy*

Abstract: In this talk, we will report our recent progress on the combinatorial approaches to the exponent problem and the loop spaces in unstable homotopy theory.