Youden's Index and the Likelihood Ratio Positive in Diagnostic Testing

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Keywords

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Summary

We refer to a recent letter to the editor by Hughes [1] and show that, despite existing similarities between Youden's index and the log-likelihood ratio positive, important differences between these two measures remain to exist which can play an important difference in clinical practice.

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Dear Editor,

In a recent letter to the Editor [1], it was pointed out that there exists a close relationship between the Youden index $J = P(T+|D+) + P(T-|D-) - 1$, sensitivity plus specificity of a diagnostic test T minus 1, and the likelihood ratio positive $LR^+ = P(T+|D+)/P(T+|D-)$, the ratio of sensitivity to the false positive rate of a diagnostic test T. Here D stands for the disease of interest and +/– indicate the presence or absence of the condition. We note that $J = P(T+|D+) - P(T+|D-)$ can be written as the difference between sensitivity and false positive rate. In the interesting note Hughes [1] uses Shannon's information principle to point out that the Youden index and the likelihood ratio positive are monotonically related. One might also argue in a more direct way to see the close connection between the two measures. Consider a first-order Taylor expansion of the logarithmic function $\log(x) \approx \log(1) + (x - 1) = x - 1$ around 1 and apply this to the likelihood ratio positive:

$$\log LR^+ = \log P(T+|D+) - \log P(T+|D-) \approx P(T+|D+) - 1 - [P(T+|D-) - 1] = J.$$

However, the above result is only an approximation and the approximation will typically be good for values of sensitivity close to 1 and values of specificity close to 0, and it is the latter which we would like to see taking much higher values in practice. A better view of the situation is provided in ▶Figure 1 which shows the difference between the log-likelihood positive and Youden's index as function of sensitivity and false positive rate (FPR = 1 – specificity). The best agreement is achieved along the diagonal which corresponds to the lower bound of diagnostic accuracy. Mostly we would be interested in the region close to the upper left corner where the diagnostic accuracy is high. Here the agreement between both measures is poor. In addition, sensitivity and specificity are typically inversely related as ▶Figure 2 shows for two normal background populations (healthy and diseased). Hence only certain pathways in the sensitivity – false positive rate diagram are possible: the so-called ROC curves which typically path through areas of low agreement between log likelihood ratio and Youden index. Fin-

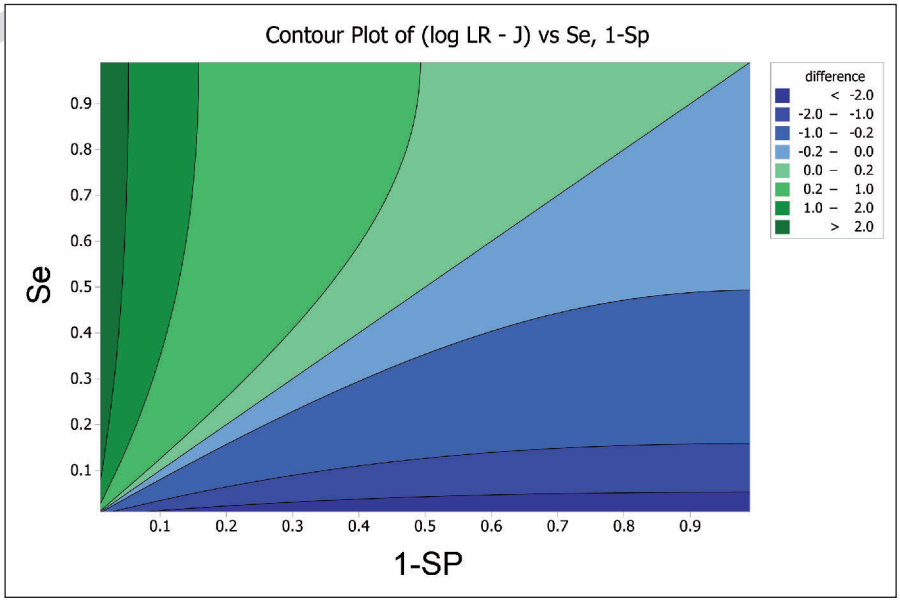

Figure 1 Contour plot of the difference between the log-likelihood ratio positive and Youden's index

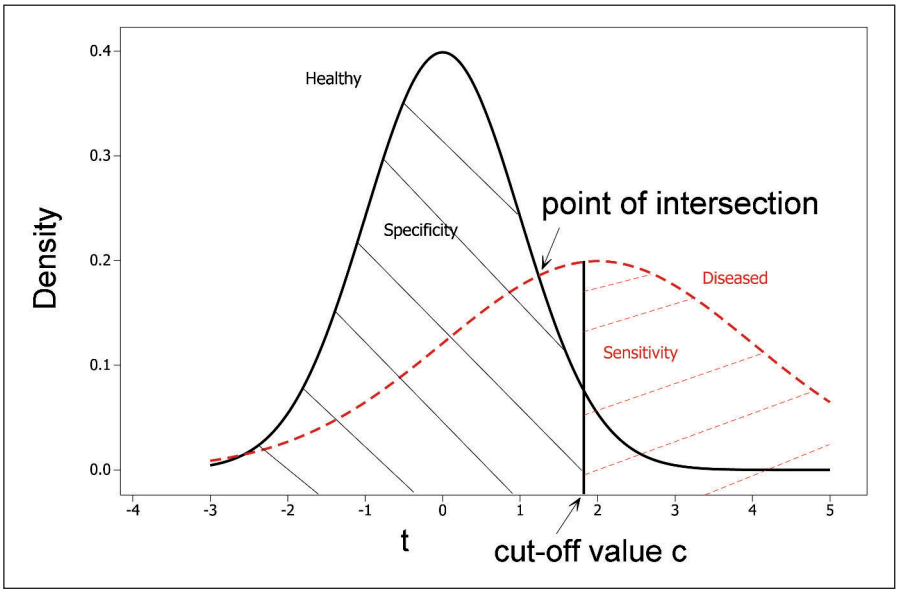

Figure 2 Sensitivity and specificity as a function of a cut-off value for a normal healthy and a normal diseased population

ally, we point out a further important difference between the two measures.

Whereas Youden's index performs well in finding a "best" cut-off value (maximizing Youden's index as a function of the cut-off), the log-likelihood ratio positive is less useful as it frequently leads to clinically improper "best" cut-offs. This is shown in ▶Figure 3 and ▶Figure 4. In particular, ▶Figure 4 shows that the largest value of the log-likelihood positive would be reached on the boundary which is clinically useless. This point has been emphasized in [2].

In summary, although there are certain similarities between Youden's index and the log-likelihood positive, important differences remain and these can play a considerable role depending on the scenario in which either index is used.

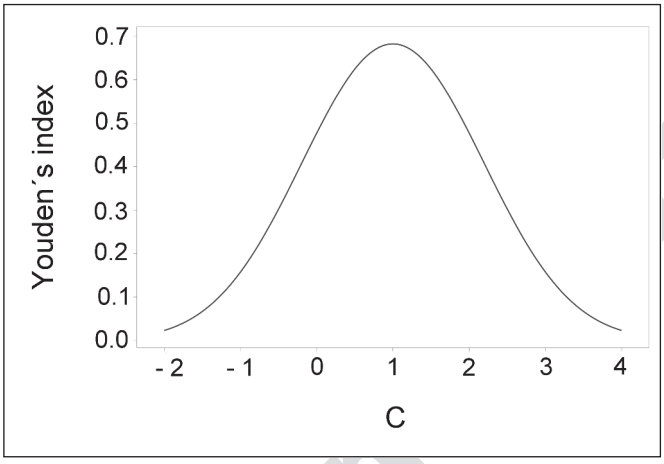

Figure 3 Youden's index as a function of a cut-off value c for two normal distributions with equal unit variance and mean difference of 2

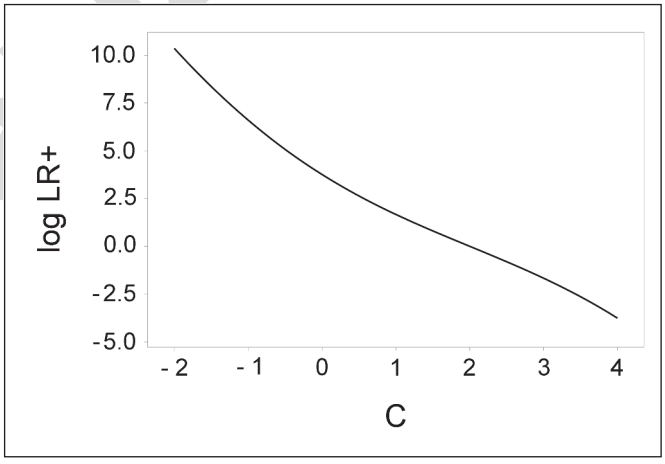

Figure 4 Log-likelihood positive as a function of a cut-off value c for two normal distributions with equal unit variance and mean difference of 2

References

1. Hughes G. Youden's index and the Weight of Evidence. Methods Inf Med 2015; 54: 198–199.
2. Böhning D, Holling H, Patilea V. A Limitation of the Diagnostic-Odds Ration in Determining an optimal Cut-Off Value for a Continuous Diagnostic Test. Statist Meth Med Res 2011; 20: 541–550.