

# Evaluating the accuracy of diagnostic systems by means of meta-analysis

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## Introduction and Background of Diagnostic Setting

## The SROC and the Littenberg-Moses Approach

## SROC-Modelling

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## Simulation Study

## Application to BNP Meta-Analysis

## Incorporating Heterogeneity by Means of a Mixed Model

## Often studies are done in medicine or psychology to determine:

**discriminatory ability** of a diagnostic test to separate people

- ▶ **with** a specific disease (or condition)
- ▶ from those **without**

in fact, diagnostic systems are all around us!

## Measures of Diagnostic Accuracy

- ▶ **Specificity:**  $P(T - | D-) = 1 - u$   
Probability of a negative test result for a healthy person
- ▶ **Sensitivity:**  $P(T + | D+) = p$   
Probability of a positive test result for a diseased person

## Estimating Diagnostic Accuracy

- ▶ **Specificity:**  $P(\widehat{T-} | D-) = 1 - \hat{u} = \frac{n-x}{n}$   
where  $x$  are the number of false-positives out of  $n$  healthy individuals,  $n - x$  are the true-negatives
- ▶ **Sensitivity:**  $P(\widehat{T+} | D+) = \hat{p} = \frac{y}{m}$   
where  $y$  are the number of true-positives out of  $m$  diseased individuals,  $y - m$  are the false-negatives

## Frequently available:

- ▶ a variety of diagnostic studies
- ▶ providing diagnostic measures

$x_i, n_i$  (specificity)

$y_i, m_i$  (sensitivity)

- ▶ for  $i = 1, \dots, k$
- ▶ leading to the field of **meta-analysis**

# An Example: Meta-Analysis of Diagnostic Accuracy of Natriuretic Peptides for Heart Failure

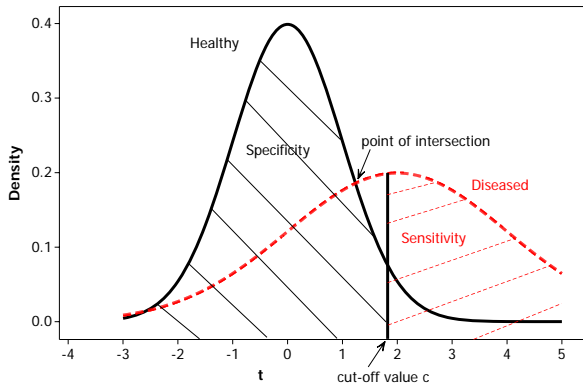
- ▶ diagnosis of heart failure is difficult
- ▶ overdiagnosis and underdiagnosis is occurring
- ▶ natriuretic peptides have been proposed as a diagnostic test
- ▶ meta-analysis provided by Doust *et al.* (2004) for brain natriuretic peptide (BNP)
- ▶ restriction on studies that use left ventricular ejection fraction of 40% or less as gold standard

# Data of Meta-Analysis on Diagnostic Accuracy of BNP for Heart Failure

	diseased		healthy		
study	$y(\text{TP})$	$m - y(\text{FN})$	$n - x(\text{TN})$	$x(\text{FP})$	$n + m$
Bettenc. 2000	29	7	46	19	101
Choy 1994	34	6	22	13	75
Valli 2001	49	9	78	17	153
Vasan 2002a	4	6	1612	85	1707
Vasan 2002b	20	40	1339	71	1470
Hutcheon 2002	29	2	102	166	299
Landray 2000	26	14	75	11	126
Smith 2000	11	1	93	50	155

# The Cut-off Value Problem

- ▶ Why not proceed with the **available armada of meta-analysis methods**?
- ▶ continuous or ordered categorical test uses **cut-off value**



## The Cut-off Value Problem

- ▶ sensitivities and specificities from different studies **not comparable**
- ▶ different values for sensitivity and specificity might be due to different **diagnostic accuracy** or **different cut-off value**
- ▶ cut-off problem introduces **bias of unknown direction and size**

## The SROC-diagram for meta-analytic situations

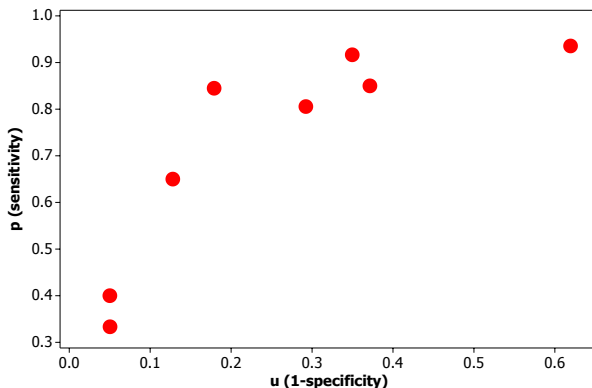
- ▶ Consider the pairs (sensitivity, 1-specificity) estimated by

$$(\hat{p}_i, \hat{u}_i) = (y_i/m_i, x_i/n_i)$$

for  $i = 1, \dots, k$

- ▶ include them in a ROC diagram
- ▶ it is called **summary** ROC because the points relate to **different studies** instead of **different cut-off values**

## SROC-diagram for MA of BNP and Heart Failure



## Few Comments on Littenberg-Moses

### The DS-Equation

Littenberg and Moses (1993) suggest to fit

$$D = \alpha + \beta S$$

and reconstruct the SROC-curve from fitted values of  $\alpha$  and  $\beta$   
where

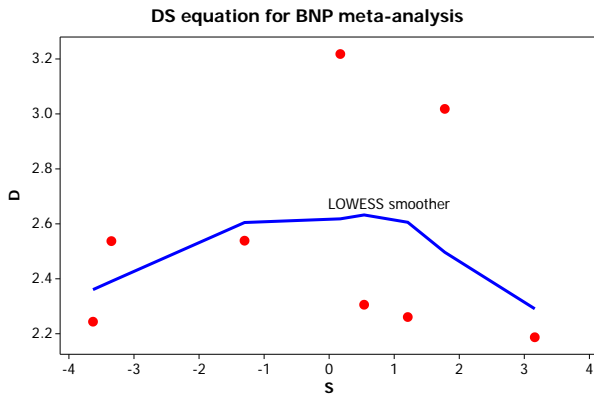
- ▶  $D = \log DOR = \log \frac{p}{1-p} - \log \frac{u}{1-u}$
- ▶  $S = \log \frac{p}{1-p} + \log \frac{u}{1-u}$

### Interpretation:

- ▶  $\alpha$  is **summary log-DOR**
- ▶ adjusted by means of  $S$  for potential **cut-off value effect**

## Problem with Littenberg-Moses:

- ▶ DS-Equation almost never met in practice



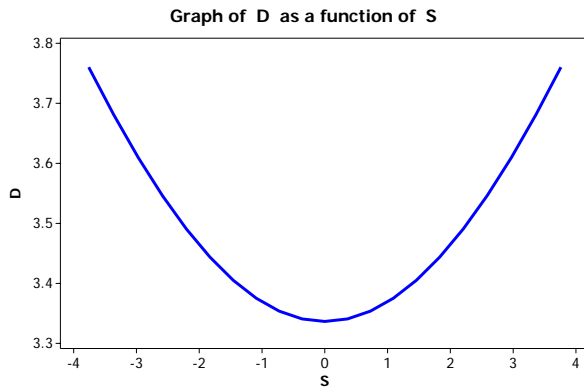
## Explanation?

### theoretical situation:

- ▶ healthy:  $T \sim N(0, 1)$ , so  $P(T < c | \text{healthy}) = \Phi(c)$
- ▶ diseased  $T \sim N(2, 1)$ , so  $P(T \geq c | \text{diseased}) = 1 - \Phi(c - 2)$

### dependency of $D$ on $S$ ?

- ▶ cut-off varies:  $c = 0, 0.1, \dots, 2$
- ▶
- ▶ How is the relation between  $D$  and  $S$ ?



## Modelling of the SROC-diagram

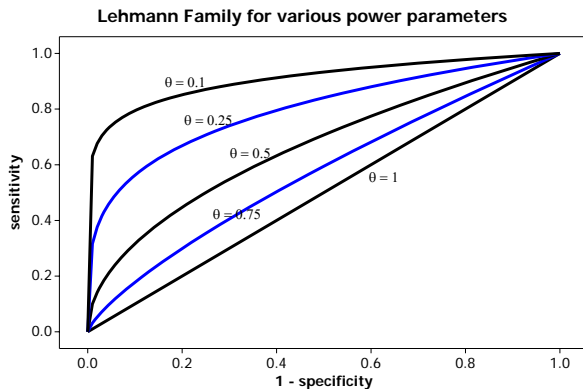
- ▶ Consider the **Lehmann family** for  $\theta > 0$  and  $i = 1, \dots, k$  (Le 2006):

$$p_i = u_i^\theta$$

- ▶ or as a simple slope-only model

$$\log p_i = \theta \log u_i$$

- ▶ note model has **one parameter of interest**  $\theta$  and  $k$  **nuisance parameters**  $u_1, \dots, u_k$
- ▶ note that  $\theta$  represents the **diagnostic power** whereas the nuisance parameter captures heterogeneity in the specificities



# Inference

- ▶ consider the product-binomial likelihood as the joint distribution of  $Y_i$  and  $X_i$  for the  $i$ -th study (index is suppressed for notational convenience)

$$\binom{m}{y} p^y (1-p)^{m-y} \times \binom{n}{x} u^x (1-u)^{n-x}$$

- ▶ which we replace by the normal approximation for  $\log Y_i$  and  $\log X_i$

$$\frac{1}{\sqrt{2\pi}s^2} \exp\left\{-\frac{1}{2} \frac{(\log y - \log(mp))^2}{s^2}\right\} \\ \times \frac{1}{\sqrt{2\pi}t^2} \exp\left\{-\frac{1}{2} \frac{(\log x - \log(nu))^2}{t^2}\right\}$$

# Inference

- ▶ the normal approximation for  $\log Y_i$  and  $\log X_i$

$$\frac{1}{2\pi st} \exp\left\{-\frac{1}{2} \frac{(\log y - \log(mp))^2}{s^2}\right\} \times \exp\left\{-\frac{1}{2} \frac{(\log x - \log(nu))^2}{t^2}\right\}$$

- ▶ with the Taylor-series variance estimates

$$s^2 = \frac{1}{y} - \frac{1}{m} \text{ and } t^2 = \frac{1}{x} - \frac{1}{n}$$

- ▶ consider now the log-likelihood for study  $i$

$$-\frac{1}{2} \frac{(\log y - \log(mp))^2}{s^2} - \frac{1}{2} \frac{(\log x - \log(nu))^2}{t^2}$$

# Inference

- and further with setting brackets differently

$$\begin{aligned}
 & -\frac{1}{2s^2}(\log y - \log m - \log p)^2 - \frac{1}{2t^2}(\log x - \log n - \log u)^2 \\
 &= -\frac{1}{2s^2}(\underbrace{\log y - \log m}_z - \log p)^2 - \frac{1}{2t^2}(\underbrace{\log x - \log n}_w - \log u)^2 \\
 &= -\frac{1}{2s^2}(z - \overbrace{\theta \log u}^{\log p})^2 - \frac{1}{2t^2}(w - \log u)^2
 \end{aligned}$$

# Inference

- ▶ leading to the log-likelihood

$$\ell(\theta, u') = -\frac{1}{2s^2}(z - \theta u')^2 - \frac{1}{2t^2}(w - u')^2$$

- ▶ maximizing  $\ell(\theta, u')$  in  $u'$  for **fixed**  $\theta$  leads to

$$\hat{u}'_{\theta} = \frac{\theta t^2 z + s^2 w}{t^2 \theta^2 + s^2}$$

- ▶ plugging  $\hat{u}'_{\theta}$  in provides the **profile log-likelihood**

$$\ell(\theta) = \ell(\theta, \hat{u}'_{\theta}) = -\frac{1}{2s^2}(z - \theta \hat{u}'_{\theta})^2 - \frac{1}{2t^2}(w - \hat{u}'_{\theta})^2$$

# Inference

- ▶ plugging  $\hat{u}'_{\theta}$  in provides the **profile log-likelihood**

$$\ell(\theta) = \ell(\theta, \hat{u}'_{\theta}) = -\frac{1}{2s^2}(z - \theta\hat{u}'_{\theta})^2 - \frac{1}{2t^2}(w - \hat{u}'_{\theta})^2$$

$$\text{with } \hat{u}'_{\theta} = \frac{\theta t^2 z + s^2 w}{t^2 \theta^2 + s^2}$$

- ▶ ... **after some work** ... simplifies to

$$\ell(\theta) = \ell(\theta, \hat{u}'_{\theta}) = -\frac{1}{2} \frac{(z - w\theta)^2}{t^2 \theta^2 + s^2}$$

a profile log-likelihood of **remarkable simplicity**

## Why profile likelihood?

- ▶ eliminates nuisance parameter
- ▶ **two forms** of the model:

$$\log p = \theta \log u \text{ or } \log u = \frac{1}{\theta} \log p$$

- ▶ it is invariant if  $u$  or  $p$  chosen to be the nuisance parameter

$$\ell(\theta, \hat{u}'_{\theta}) = \ell(\theta, \hat{p}'_{\theta})$$

- ▶ suitable for **symmetric** regression problems

## Profile or Adjusted Profile Likelihood?

- ▶  $\ell(\theta)$  is **almost Gaussian**

$$\ell(\theta) = \ell(\theta, \hat{u}'_{\theta}) = -\frac{1}{2} \underbrace{\frac{(z - w\theta)^2}{t^2\theta^2 + s^2}}_{\sigma^2(\theta)}$$

- ▶ it differs **only** from

$$L(\theta) = -\frac{1}{2} \log \sigma^2(\theta) - \frac{1}{2} \frac{(z - w\theta)^2}{\sigma^2(\theta)}$$

**by**  $\frac{1}{2} \log \sigma^2(\theta)$

## Profile or Adjusted Profile Likelihood?

- ▶  $\frac{1}{2} \log \sigma^2(\theta)$  corresponds to the **adjustment factor**  $\hat{l}(\hat{u}_\theta)^{-1/2}$
- ▶ suggested by Cox and Reed (1987); Lee, Nelder Pawitan (2006); Murphy and van der Vaart (2000):

$$\begin{aligned}\hat{l}(\hat{u}_\theta) &= -\frac{\partial^2}{\partial u'^2} \ell(\theta, u') = \frac{\partial^2}{\partial u'^2} \left( \frac{1}{2s^2} (z - \theta \hat{u}')^2 + \frac{1}{2t^2} (w - \hat{u}')^2 \right) \\ &= \frac{t^2 \theta^2 + s^2}{s^2 t^2}\end{aligned}$$

- ▶ where, for fixed  $\theta$ ,  $\hat{l}(\hat{u}_\theta)$  is the **observed Fisher information**  $\hat{l}(u)$  evaluated at  $\hat{u}_\theta$

## Profile or Adjusted Profile Likelihood?

- ▶ as can be seen directly from above that

$$-\frac{1}{2} \log[\hat{l}(\theta)] + \ell(\theta) = L(\theta)$$

- ▶ providing an **excellent** justification of the adjusted profile likelihood

## Full Sample Profile Likelihoods

for a sample of  $k$  studies

- ▶ we have the **full-sample profile** log-likelihood

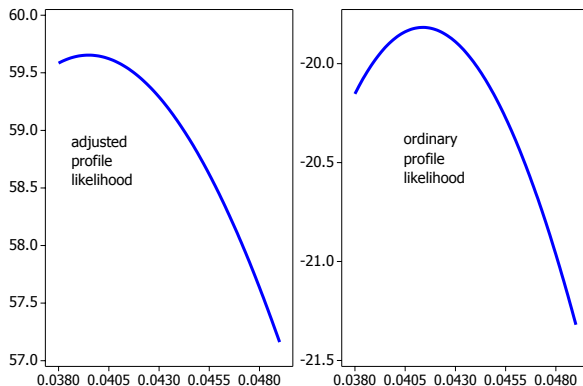
$$\ell(\theta) = - \sum_i \frac{1}{2} \frac{(z_i - w_i \theta)^2}{\sigma_i^2(\theta)}$$

- ▶ and the **full-sample adjusted profile** log-likelihood

$$L(\theta) = - \sum_i \frac{1}{2} \log \sigma_i^2(\theta) - \sum_i \frac{1}{2} \frac{(z_i - w_i \theta)^2}{\sigma_i^2(\theta)}$$

where  $\sigma_i^2(\theta) = t_i^2 \theta^2 + s_i^2$ .

## Ordinary and Adjusted Profile Likelihoods



## Estimation: Maximum Profile Likelihood

- score for the **ordinary profile likelihood**

$$\begin{aligned}\frac{d}{d\theta}\ell(\theta) &= -\frac{d}{d\theta} \sum_i \frac{1}{2} \frac{(z_i - w_i\theta)^2}{\sigma_i^2(\theta)} \\ &= \sum_i \frac{(z_i - w_i\theta)w_i}{\sigma_i^2(\theta)} + \frac{1}{2} \frac{(z_i - w_i\theta)^2 \sigma_i^2(\theta)'}{\sigma_i^4(\theta)}\end{aligned}$$

- and the score for the **adjusted profile likelihood**

$$\begin{aligned}\frac{d}{d\theta}L(\theta) &= \frac{d}{d\theta}\ell(\theta) - \frac{d}{d\theta} \sum_i \frac{1}{2} \log \sigma_i^2(\theta) \\ &= \frac{d}{d\theta}\ell(\theta) - \frac{1}{2} \sum_i \frac{\sigma_i^2(\theta)'}{\sigma_i^2(\theta)}\end{aligned}$$

## Estimating Equation Approach

- ▶ suggestion: fix  $\theta$  in  $\sigma_i^2(\theta)$  and maximize the Gaussian loss in  $\theta$ :

$$-\sum_i \frac{(z_i - w_i\theta)^2}{\sigma_i^2(\theta)}$$

- ▶ or solve the estimating equation

$$\sum_i \frac{(z_i - w_i\theta)w_i}{\sigma_i^2(\theta)} = 0$$

- ▶ leading to the **iterative reweighted least-squares** approach:

$$\theta = \frac{\sum_i z_i w_i / \sigma_i^2(\theta)}{\sum_i w_i^2 / \sigma_i^2(\theta)}$$

## Estimating Equation Approach

- ▶ neither ordinary nor adjusted profile likelihood is equivalent to IWLS
- ▶ look at the score for the **adjusted profile likelihood**

$$\begin{aligned}
 &= \sum_i \frac{(z_i - w_i \theta) w_i}{\sigma_i^2(\theta)} + \frac{1}{2} \frac{\overbrace{(z_i - w_i \theta)^2}^{\sigma_i^2(\theta)} \sigma_i^2(\theta)'}{\sigma_i^4(\theta)} - \frac{1}{2} \frac{\sigma_i^2(\theta)'}{\sigma_i^2(\theta)} \\
 &\approx \sum_i \frac{(z_i - w_i \theta) w_i}{\sigma_i^2(\theta)}
 \end{aligned}$$

- ▶ close to **estimating equation** approach

# Simulation Study

- ▶ previous analysis suggests: profile and adjusted profile likelihood inference differs
- ▶ but how much? Look at **Bias and variance!**
- ▶ how **valid** are the second derivate approximations of the true variances for both likelihoods ?

## Simulation Study: Design

for  $i = 1, \dots, k = 10$ :

1.  $u_i \sim U[0.05, .5]$
2. use model:  $p_i = u_i^\theta$  for  $\theta = 0.1, 0, 2, 0.3$
3.  $n_i, m_i \sim Po(100)$  or  $n_i, m_i \sim Po(10)$  (sparsity case)
4.  $Y_i \sim Bin(p_i, m_i)$  and  $X_i \sim Bin(u_i, n_i)$
5. determine various estimators of  $\theta$
6. replicate this process 1,000 times

## Simulation Study: Results

**Table:** *Mean and Variance for Profile (PMLE), Adjust Profile (APMLE) and Iterative Weighted Least Squares (IWLS) Estimator*

estimator for $\theta = 0.1$	$E(\hat{\theta})$	$SE(\hat{\theta})$	$\widehat{SE}(\hat{\theta})$
$E(n_i) = E(m_i) = 100$			
IWLS	0.0961	0.0104	-
PMLE	0.0977	0.0104	0.0119
APMLE	0.0960	0.0101	0.0117
$E(n_i) = E(m_i) = 10$			
IWLS	0.0899	0.0291	-
PMLE	0.0981	0.0313	0.0561
APMLE	0.0812	0.0260	0.0468

## Simulation Study: Results

**Table:** *Mean and Variance for Profile (PMLE), Adjust Profile (APMLE) and Iterative Weighted Least Squares (IWLS) Estimator*

estimator for $\theta = 0.2$	$E(\hat{\theta})$	$SE(\hat{\theta})$	$\widehat{SE}(\hat{\theta})$
$E(n_i) = E(m_i) = 100$			
IWLS	0.1959	0.0153	-
PMLE	0.1988	0.0153	0.0194
APMLE	0.1955	0.0151	0.0191
$E(n_i) = E(m_i) = 10$			
IWLS	0.1722	0.0499	-
PMLE	0.1917	0.0536	0.0838
APMLE	0.1597	0.0442	0.0654

# Simulation Study: Results

**Table:** *Mean and Variance for Profile (PMLE), Adjust Profile (APMLE) and Iterative Weighted Least Squares (IWLS) Estimator*

estimator for $\theta = 0.3$	$E(\hat{\theta})$	$SE(\hat{\theta})$	$\widehat{SE}(\hat{\theta})$
$E(n_i) = E(m_i) = 100$			
IWLS	0.2953	0.0210	-
PMLE	0.3004	0.0211	0.0262
APMLE	0.2953	0.0208	0.0255
$E(n_i) = E(m_i) = 10$			
IWLS	0.2693	0.0694	-
PMLE	0.3011	0.0742	0.1137
APMLE	0.2517	0.0622	0.0869

## Simulation Study: Results for small $n$ but large $k$

Table: Mean and Variance for Profile (PMLE), Adjust Profile (APMLE) and Iterative Weighted Least Squares (IWLS) Estimator  
 $k=100$

estimator for $\theta = 0.3$	$E(\hat{\theta})$	$SE(\hat{\theta})$	$\widehat{SE}(\hat{\theta})$
$E(n_i) = E(m_i) = 20$			
IWLS	0.2753	0.0153	-
PMLE	0.2970	0.0156	0.0189
APMLE	0.2718	0.0143	0.0164

## Simulation Study: Conclusions

### large $n_i, m_i$

- ▶ all three estimators behave similar
- ▶ minimal gain in efficiency with APMLE
- ▶ Fisher information estimate a bit conservative for variance estimation

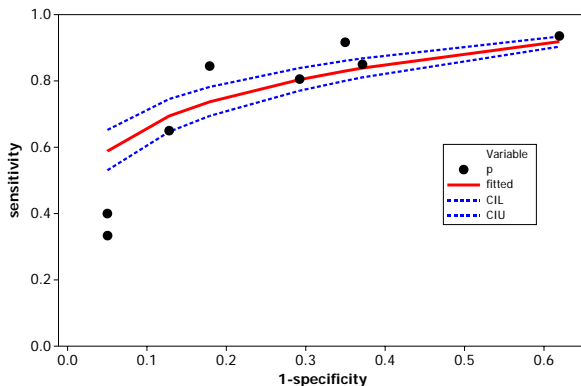
### small $n_i, m_i$

- ▶ ordinary PMLE less biased
- ▶ APMLE more efficient
- ▶ Fisher information estimate overestimates variance of PMLE and APMLE

## Application to BNP Meta-Analysis

- ▶ APMLE for  $L(\theta)$  provides  $\hat{\theta} = 0.1774$
- ▶ PMLE for  $\ell(\theta)$  provides  $\hat{\theta} = 0.1802$
- ▶ and IWLS gives  $\hat{\theta} = 0.1755$

## Observed and Fitted Lehmann Model



## Goodness-of-Fit

- ▶ since  $E(Z_i) = \theta \log u_i$  and  $E(W_i) = \log u_i$
- ▶ it follows

$$E(Z_i - \theta W_i) = 0$$

- ▶ also  $\text{Var}(Z_i) = s_i^2$  and  $\text{Var}(\theta W_i) = \theta^2 t_i^2$
- ▶ hence

$$\text{Var}(Z_i - \theta W_i) = s_i^2 + \theta^2 t_i^2$$

- ▶ so that

$$\frac{Z_i - \theta W_i}{\sqrt{s_i^2 + \theta^2 t_i^2}} \sim N(0, 1)$$

## Goodness-of-Fit

$\chi^2$  – statistic

$$\chi_{k-1}^2 = \sum_{i=1}^k \frac{(Z_i - \hat{\theta}W_i)^2}{s_i^2 + \hat{\theta}^2 t_i^2}$$

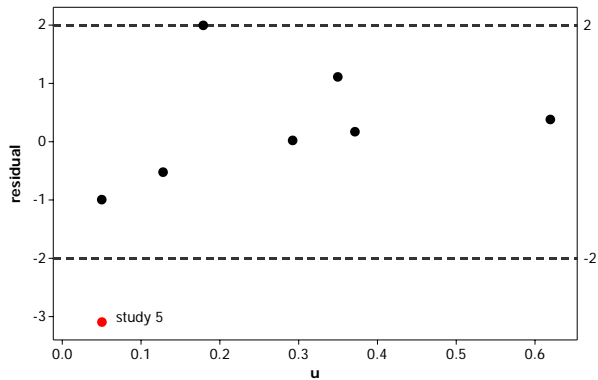
### BNP meta-analysis

based upon **all** 8 studies:  $\chi_7^2 = 16.23$  and  $P = 0.0231$

based upon 7 Studies (without **study 5**):  $\chi_6^2 = 6.66$  and  $P = 0.4655$  since plot of residuals:

$$\frac{Z_i - \hat{\theta}W_i}{\sqrt{s_i^2 + \hat{\theta}^2 t_i^2}}$$

provides evidence that study 5 is source of heterogeneity



# Incorporating Heterogeneity

- ▶ known: heterogeneity is **frequent** in meta-analytic settings
- ▶ often required to incorporate heterogeneity into the model building process
- ▶ in addition: comparative issues are confounded by forms of heterogeneity
- ▶ this will be illustrated with an example from **alcohol use disorder** identification

# Meta-Analysis on Diagnostic Accuracy of the Alcohol Use Disorder Identification Test (AUDIT)

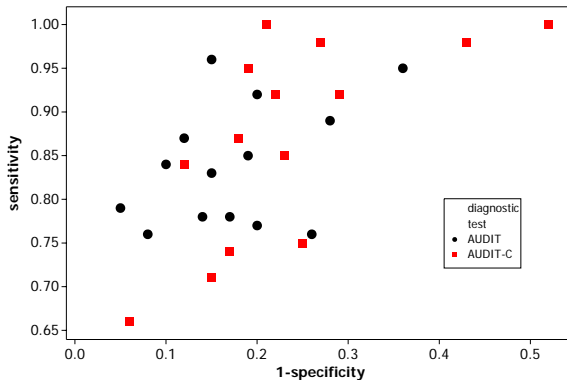
- ▶ Kristen *et al.* (2008): *Are 3 Questions enough to detect unhealthy alcohol use?*
- ▶ 10-item AUDIT and abbreviated 3-item version AUDIT-C to detect unhealthy alcohol use
- ▶ Purpose: is AUDIT-C is as accurate as the full AUDIT
- ▶ MA of 14 studies for the AUDIT as follows

	alcohol disorder		no disorder		
study	$y(\text{TP})$	$m - y(\text{FN})$	$n - x(\text{TN})$	$x(\text{FP})$	$n + m$
1	48	7	738	101	894
2	138	39	1506	309	1992
3	24	5	173	31	233
4	37	2	227	127	393
5	137	12	936	234	1319
6	73	13	127	30	243
7	53	14	508	27	602
8	571	180	5707	496	6954
9	54	10	172	19	255
10	148	44	2687	672	3551
11	143	18	334	130	625
12	47	13	464	76	600
13	34	1	65	12	112
14	154	49	261	92	555

# Meta-Analysis on Diagnostic Accuracy of the Alcohol Use Disorder Identification Test (AUDIT)

- ▶ the MA also includes studies on the AUDIT-C
- ▶ MA of 14 studies for the AUDIT-C as follows

	alcohol disorder		no disorder		
study	$y(\text{TP})$	$m - y(\text{FN})$	$n - x(\text{TN})$	$x(\text{FP})$	$n + m$
1	47	9	738	101	894
2	126	51	1543	272	1992
3	19	10	192	12	233
4	36	3	276	78	393
5	130	19	959	211	1319
6	84	2	89	68	243
7	67	0	423	112	602
8	751	0	2977	3226	6954
9	59	5	136	55	255
10	142	50	2788	571	3551
11	137	24	358	107	625
12	57	3	437	103	600
13	34	1	56	21	112
14	152	51	264	88	555



## ROC curves of AUDIT and AUDIT-C:

- ▶ no clear **evidence** for either/or
- ▶ highly **unsmooth**
- ▶ evidence of **heterogeneity**

## How well is the Lehmann model supported in the data?

scatterplot of  $\log p$  vs.  $\log u$ :

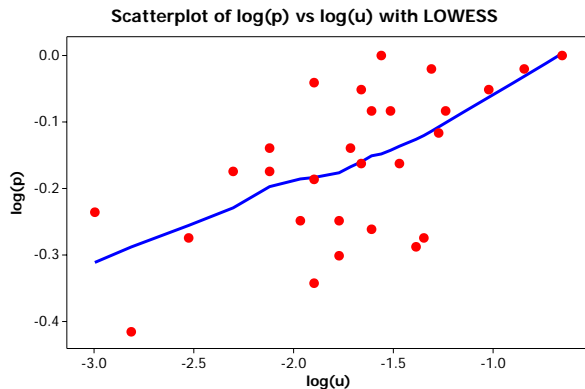
since

$$p = u^\theta$$

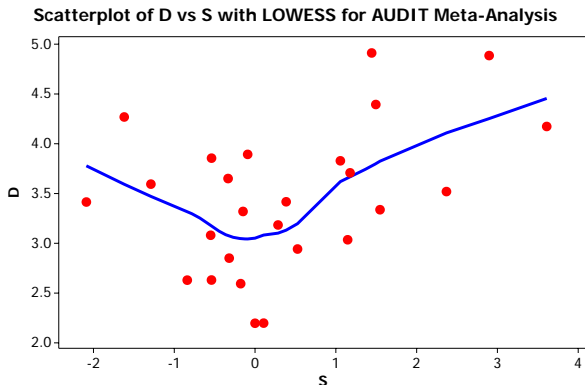
is equivalent to

$$\log p = \theta \log u$$

we look at the linearity of scatterplot  $\log p$  vs.  $\log u$



## Littenberg-Moses would be problematic here



## Extension to a Random Effect Model

for a sample of  $k$  studies

- ▶ the **full-sample adjusted profile** log-likelihood is

$$L(\theta) = - \sum_{i=1}^k \frac{1}{2} \log \sigma_i^2(\theta) - \sum_{i=1}^k \frac{1}{2} \frac{(z_i - w_i \theta)^2}{\sigma_i^2(\theta)}$$

where  $\sigma_i^2(\theta) = t_i^2 \theta^2 + s_i^2$

- ▶ this suggests to consider  $Z_i$  conditional upon  $W_i$

$$Z_i = w_i \theta + \epsilon_i$$

- ▶ so that

$$Z_i | w_i \sim N(w_i \theta, \sigma_i^2(\theta))$$

## Extension to a Random Effect Model

- ▶ this can be extended by a further **random effect**  $\delta_i$ , independent of  $\epsilon_i$ , with  $E(\delta_i) = 0$  and  $Var(\delta_i) = \tau^2$

$$Z_i = w_i\theta + \delta_i + \epsilon_i$$

- ▶ so that

$$Z_i|w_i \sim N(w_i\theta, \sigma_i^2(\theta) + \tau^2)$$

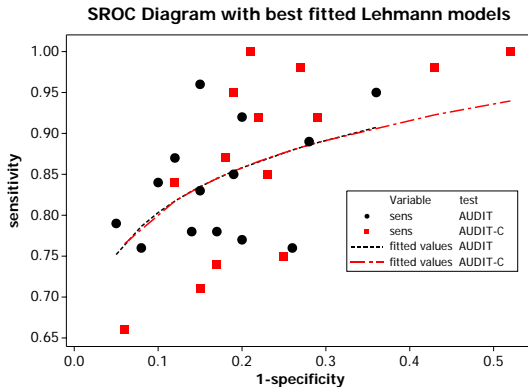
- ▶ the **full-sample adjusted profile** log-likelihood with **random effect** is

$$L(\theta, \tau^2) = - \sum_{i=1}^k \frac{1}{2} \log[\sigma_i^2(\theta) + \tau^2] - \sum_{i=1}^k \frac{1}{2} \frac{(z_i - w_i\theta)^2}{\sigma_i^2(\theta) + \tau^2}$$

where  $\sigma_i^2(\theta) = t_i^2\theta^2 + s_i^2$

# Analysis of Diagnostic Accuracy for AUDIT Meta-Analysis Data

Test	$\hat{\theta}$	$\hat{\tau}^2$	$\chi^2$	P-value
— homogeneity —				
AUDIT	0.09836	-	54.45	< 0.001
AUDIT-C	0.09145	-	3,200	< 0.001
— heterogeneity (mixed model) —				
AUDIT	0.09402	0.00457	13.07	0.36
AUDIT-C	0.09516	0.00825	13.85	0.31



# Analysis of Diagnostic Accuracy for AUDIT Meta-Analysis Data: Interpretation

MA shows that

- ▶ AUDIT and AUDIT-C have similar diagnostic accuracy
- ▶ heterogeneity for AUDIT-C is considerably larger

## Further Work in Progress

- ▶ general linear model approach to include **observed heterogeneity** in form of **covariates**
- ▶ nonparametric mixture approach to model **unobserved heterogeneity**
- ▶ **classification** of studies into different components of **homogeneous diagnostic accuracy**