MATH3085/6143 Survival Models – Worksheet 6

1. i) Describe the differences between the central exposed to risk and the initial exposed to risk.

The following data come from an investigation of the mortality of participants in a dangerous sport during the calendar year 2005.

Age x	No. of lives aged x last birthday on 1 January 2005	No. of lives aged x last birthday on 1 January 2006	No. of deaths during 2005 of persons aged x last birthday at death
22	150	160	20
23	160	155	25

ii) Estimate the initial exposed to risk at ages 22 and 23. Hence estimate q_{22} and q_{23}.

iii) Suppose that in this investigation, instead of aggregate data, we had individual-level data on each person's date of birth, date of death, and date of exit from observation (if exit was for reasons other than death). Explain how you would calculate the initial exposed-to-risk for lives aged 22 years last birthday.

2. i) Clearly explain what is meant by the census approximation of the central exposed to risk. What is the principal assumption underlying the census approximation?

A mortality investigation bureau has collected the following information on number of policies in-force each year from different companies:

Age	Year	Company A	Company B	Company C
44	2010	5868	3928	9176
	2011	5883	3946	9176
	2012	5909	3938	9187
45	2010	5928	3939	9198
	2011	5920	3921	9148
	2012	5911	3930	9136
46	2010	5977	3969	9237
	2011	5993	3973	9252
	2012	5988	4018	9240

However, the bureau did not specify its data requirement precisely and consequently has received inconsistent submissions from different companies:

- Company A has provided in-force policy data as at the beginning of each year (i.e. 1 January) using age nearest birthday;
- Company B has provided in-force policy data as at the financial year closing date (i.e. 31 March in each year) using age last birthday; and
- Company C has provided in-force policy data as at the end of each year (i.e. 31 December) using age next birthday.

ii) Based on the information above, determine the contribution to central exposed to risk for aged 45 last birthday for the calendar year 2011 from the data available for each of the companies.

3. A pension fund is developing a new class of business for male lives. It has conducted a study of mortality among lives it believes represent this new business. It wishes to compare the data with a previous table to see if they are similar. It decides to try the PMA92C20 tables (male pensioner mortality projected to 2020) table first.

Age, x	Observed rates, $\hat{m}_x$	PMA92C20 rates, m_x^S	Exposed to risk
70	0.0167	0.012661	1200
71	0.0150	0.014783	1194
72	0.0171	0.017204	973
73	0.0187	0.019956	956
74	0.0245	0.023072	912
75	0.0402	0.026587	845
76	0.0561	0.030357	820
77	0.0623	0.034962	369
78	0.0552	0.039899	489
79	0.0640	0.045390	500

i) Carry out an overall test of the goodness of fit of the PMA92C20 rates to the observed rates.

ii) List three deviations between observed and standard rates which the test you conducted in (i) may not detect.

4. The mortality experience of the male population of a region of the United Kingdom is to be compared with a set of standard mortality rates. The following is an extract from the results.

Age, x	Observed number of deaths, d_x	Central exposed to risk, E_x^C	Standard mortality rate, m_x^S	Expected number of deaths, $E_x^C m_x^S$
14	3	12800	0.00038	4.86
15	8	15300	0.00043	6.58
16	5	12500	0.00048	6.00
17	14	15000	0.00053	7.95
18	17	16500	0.00059	9.74
19	9	10100	0.00066	6.67
20	15	12800	0.00074	9.47
21	10	13700	0.00083	11.37
22	10	11900	0.00093	11.07
Total	91			73.71

Test whether the data conform to the standard mortality rates using:

i) A test of overall goodness of fit.

ii) A test of systematic bias.

5. A large insurance company has performed graduation on the mortality experience of part of its business, by a linear regression of observed (log) rates against historical (log) rates. The original data and the graduated rates are as follows.

Age	Observed number of deaths	Exposed to risk	Graduated rates
40	4	1284	0.00240
41	4	2038	0.00266
42	12	1952	0.00297
43	7	2158	0.00332
44	11	2480	0.00371
45	7	1456	0.00415
46	12	2100	0.00464
47	16	1866	0.00519
48	15	1989	0.00577
49	10	1725	0.00642

i) Test this graduation for overall goodness of fit.

ii) Investigate the graduation for individual outliers.