MATH3085/6143 Survival Models – Worksheet 5

1. The table below is extracted from English Life Table 15 (females).

Age x (years)	ℓ_x	$\mathring{e}_x$
0	100000	78.96
10	99172	69.61
20	98957	59.75
30	98617	49.94
40	97952	40.24

   i) Calculate ${}_{10}q_0$.

   ii) Calculate the number of woman-years lived at ages over 30 and 40 years respectively among 100000 women born.

   iii) a) Calculate the number of woman-years lived between exact ages 30 and 40 years by those who die between 30 and 40 years.

   b) Hence calculate the average age at death of those who die between exact ages 30 and 40 years.

2. Below is an extract from English Life Table 15 (females), which relates to mortality in England and Wales during the period 1990-1992.

Age x (years)	ℓ_x	Age x (years)	ℓ_x
0	100000	20	98957
1	99368	25	98797
5	99243	30	98617
10	99172	35	98359
15	99098	40	97952

The complete life expectancy at age 40 years according to this life table was known to be 40.237 years.

   i) Stating any assumptions you make, define and determine the values of the life table quantities, ${}_5d_{30}$, ${}_4q_1$, and ${}_5L_{30}$.

   ii) What is the probability that a woman aged 20 years will die before she attains the age of 40 years?

   iii) Compute the curtate and complete expectation of life at birth.

3. A mortality investigation followed a sample of 10000 persons from their 40^{th} birthday until their death, or until they attained the age of 50 years. There were no withdrawals from the investigation for reasons other than death. The table below gives the number of persons alive at various ages.

Age x (years)	ℓ_x
40	10000
45	9883
50	9692

Assuming a uniform distribution of deaths within each five-year age group, estimate:

   i) The number of persons who survive to exact age 41 years.

   ii) The probability that a life aged exactly 40 years will die between ages 44 and 46 years.

4. An investigation took place into the mortality of persons between exact ages 60 and 61 years. The table below gives an extract from the results. For each person it gives the age at which they were first observed, the age at which they ceased to be observed and the reason for their departure from observation.

Person	Age at entry (years/months)	Age at exit (years/months)	Reason for exit
1	60/0	60/6	withdrew
2	60/1	61/0	survived to 61
3	60/1	60/3	died
4	60/2	61/0	survived to 61
5	60/3	60/9	died
6	60/4	61/0	survived to 61
7	60/5	60/11	died
8	60/7	61/0	survived to 61
9	60/8	60/10	died
10	60/9	61/0	survived to 61

i) Estimate q_{60} using a Binomial model, stating any assumptions you make.

ii) Hence estimate μ_{60}, stating any assumptions you make.

5. An investigation took place into the mortality of pensioners. The investigation began on 1 January 2003 and ended on 1 January 2004. The table below provides the data collected in this investigation for 8 lives.

Date of birth	Date of entry	Date of exit	Exit was due to death (1) or not (0)
1 April 1932	1 January 2003	1 January 2004	0
1 October 1932	1 January 2003	1 January 2004	0
1 November 1932	1 March 2003	1 September 2003	1
1 January 1933	1 March 2003	1 June 2003	1
1 January 1933	1 June 2003	1 September 2003	0
1 March 1933	1 September 2003	1 January 2004	0
1 June 1933	1 January 2003	1 January 2004	0
1 October 1933	1 June 2003	1 January 2004	0

By assuming a constant force of mortality, μ_{70}, between exact ages 70 and 71, estimate μ_{70} using a Poisson model and the data for the 8 lives in the table.

6. An investigation studied the mortality of individuals aged between 60 and 61 years (last birthday). The investigation began on 1 January 2006 and ended on 1 January 2007. The following table gives details of 8 lives involved in the investigation. (You may assume that all months have 30 days)

Life	Date of 60^{th} birthday	Date of death (if died during 2006)
1	1 February 2005	
2	1 March 2005	1 December 2006
3	1 August 2005	
4	1 September 2005	
5	1 February 2006	1 April 2006
6	1 April 2006	
7	1 June 2006	
8	1 September 2006	1 November 2006

i) Assuming a binomial model with a uniform distribution of deaths between the ages of 60 and 61, find an expression for the log-likelihood $\ell(q_{60})$ and hence find an equation, the solution to which is the m.l.e. for q_{60}

ii) Find the value of an alternative estimate for q_{60}, based on the initial exposed to risk E_{60}.

iii) Hence estimate the force of mortality $\mu_{60.5}$, stating any assumptions you make.

iv) Compare this estimate of $\mu_{60.5}$ with that obtained using a two-state model. Explain why the two estimates are different.