MATH3085/6143 Survival Models – Worksheet 1

1. You wish to investigate the duration of post-operative stay, T, in hospital among patients who undergo a particular type of surgery.

   i) Describe a suitable observational plan which would allow you to estimate the survival function of T, $S_T(t)$.

   ii) List three factors which might give rise to censoring according to your plan.

   iii) Suggest circumstances which might lead to the violation of the assumptions that censoring is non-informative.

2. A survival process is modelled using the hazard of failure

$$h_T(t) = \alpha + \beta t,$$

   for some constants α and β. Obtain expressions for the survival function and the density function, $S_T(t)$ and $f_T(t)$ respectively, in terms of α and β. What constraints need to be placed on α and β so that this is a proper hazard function?

3. The log-logistic distribution can be represented by the p.d.f.

$$f_T(t) = \frac{\alpha\lambda t^{\alpha-1}}{(1+\lambda t^{\alpha})^2},$$

   where $\alpha > 0$ and $\lambda > 0$ are parameters. Derive expressions for the survival function, $S_T(t)$ and the hazard function, $h_T(t)$ in terms of α and λ.

4. Derive the survival and hazard functions for the Gompertz distribution.

5. The Makeham distribution is a three-parameter extension to the Gompertz distribution, with hazard function

$$h_T(t) = \lambda + \alpha \exp(\beta t)$$

   where $\alpha > 0$, $\beta > 0$ and $\lambda > 0$ are parameters. Derive expressions for the survival function, $S_T(t)$ and the density function, $f_T(t)$ in terms of α, β and λ.

   If the Makeham model is used to model human mortality (in middle and older-ages), as an alternative to the Gompertz model, how might the extra parameter λ be interpreted?