

# A simple model of fluid flow in a scraped-surface heat exchanger

**B. R. Duffy**<sup>1</sup> **S. K. Wilson**<sup>2</sup>

Department of Mathematics, University of Strathclyde,  
Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH

and **M. E. M. Lee**<sup>3</sup>

Faculty of Mathematical Studies, University of Southampton,  
Highfield, Southampton SO17 1BJ

Scraped-surface heat exchangers (SSHEs) are widely used in the food industry to transfer heat into and out of foodstuffs in a quick and efficient manner without causing unwanted changes to the constitution, texture and appearance of the final product. A SSHE is essentially a cylindrical annulus with a heated or cooled outer wall; the foodstuff passes along the annulus, and as it does so a ‘bank’ of blades rotating with the inner wall is used to scrape it away from the outer wall, preventing fouling, and maintaining heat transfer and mixing. Commonly foodstuffs are complex fluids containing particles of varying sizes, and exhibiting non-Newtonian and heat-thinning characteristics.

Despite their widespread industrial use there is at present only a limited understanding of the fundamental mechanisms at work in SSHEs; the talk will present a simple mathematical model of aspects of the fluid behaviour within a SSHE.

In practice the gaps between the blades and the walls are fairly long and narrow. To model flow in a transverse cross section of the annulus, we use a lubrication approximation in an analysis of steady isothermal two-dimensional flow of a Newtonian fluid around a freely pivoted scraper blade in a periodic array of blades. The main conclusion is that in practice the blade tips will usually be in close contact with the wall, leading to singular behaviour of the forces in the system. We then consider how non-Newtonian fluid characteristics and slip boundary conditions at the wall can alleviate these singularities. Lastly we consider briefly thermal effects in the case when one of the walls is heated.

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<sup>1</sup>Telephone : + 44 (0)141 548 3645, Fax : + 44 (0)141 552 8657, Email : [b.r.duffy@strath.ac.uk](mailto:b.r.duffy@strath.ac.uk)

<sup>2</sup>Telephone : + 44 (0)141 548 3820, Fax : + 44 (0)141 552 8657, Email : [s.k.wilson@strath.ac.uk](mailto:s.k.wilson@strath.ac.uk)

<sup>3</sup>Telephone : + 44 (0)2380 23137, Fax : + 44 (0)2380 595147, Email : [m.e.lee@maths.soton.ac.uk](mailto:m.e.lee@maths.soton.ac.uk)