PHYS3002 - NUCLEI AND PARTICLES

Problem Sheet 2 - Due February 23, 2015

1. Use the Shell Model to determine, wherever possible, the spins and parities of the ground states of the following nuclides:

$$^{13}_{6}\mathrm{C},\ ^{14}_{6}\mathrm{C},\ ^{17}_{8}\mathrm{O},\ ^{16}_{8}\mathrm{O},\ ^{33}_{16}\mathrm{S},\ ^{31}_{15}\mathrm{P},\ ^{30}_{15}\mathrm{P},\ ^{32}_{15}\mathrm{P}.$$

   Where it is not possible to determine the spins and/or parities exactly, give the possible values that these quantities can take. [4]

2. An even-even nucleus with A=170 has a sequence of excited states with energies above the ground state:

$$\mathrm{E\,(KeV)}\quad 97,\quad 321,\quad 678,\quad 1164.$$

   Explain why we can interpret these as rotational states.
   Calculate the moment of inertia of the nucleus (in units of $\mathrm{GeV/c^2\,fm^2}$).
   Taking the nuclear radius to be $\mathrm{R} = 1.3\mathrm{A}^{1/3}$, compare this with the moment of inertia about an axis through the centre of a sphere of mass equal to the mass of the nucleus and radius equal to the radius of the nucleus. [4]

   [The moment of inertia of a sphere of mass M and radius R about an axis through its centre is $\frac{2}{5}MR^2$].

3. Identify the mode of radioactive decay (i.e. α-decay, β-decay) for the following radioactive transitions:

$$^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb},\ ^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th},\ ^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa},\ ^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{A},\ ^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca}.$$

   In the case of β-decay indicate whether an electron or a positron is emitted.
   Which of these transitions can process by electron capture? [3]

4. 1 g. of carbon from a relic found in an Egyptian tomb has a measured activity of 0.15 Bq. The ratio of $^{14}_{6}\mathrm{C}$ to $^{12}_{6}\mathrm{C}$ in living plants is 1.3×10^{-12}. The half-life of $^{14}_{6}\mathrm{C}$ is 5731 yr. How old is the relic? [3]

5. $^{238}_{92}\mathrm{U}$ has a radioactive mean lifetime of $4.5 \pm 0.1 \times 10^9$ yr. How long does one have to observe the radioactivity from 1 mg of $^{238}_{92}\mathrm{U}$ in order to be able to determine its lifetime to this accuracy? [3]

6. $^{210}_{83}\mathrm{Bi}$ has a mean lifetime of 7.2 days, decaying by β-decay into Po which in turn decays by α-emission into Pb, with a mean lifetime of 200 days. What are the atomic numbers and atomic mass numbers of the Po and Pb? If the source originally contains only Bi, after how long will the number of Pb nuclei reach a maximum? [4]

7. The binding energy of $^{240}_{94}$Pu is 1813.45 MeV, and that of $^{236}_{92}$U is 1790.40 MeV. The binding energy of $^{4}_{2}$He is 28.30 MeV.
   Calculate (to an accuracy of 10 KeV) the kinetic energy of the α-particle emitted in the decay
   $$^{240}_{94}\mathrm{Pu} \;\rightarrow\; ^{236}_{92}\mathrm{U} + \alpha.$$
   For some of these decays the α-particle is accompanied by a 50 KeV photon. Explain how this occurs and find the kinetic energy of the emitted α-particle for these events. [4]

Non-Assessed Questions

1. Discuss the evidence for magic numbers.

2. Describe the Shell Model of the nucleus, indicating the importance of spin-orbit coupling.

3. A radioactive nuclide decays with a mean lifetime τ_1, into another radioactive nuclide which in turn decays with a mean lifetime, τ_2.
   Write down the differential equations for the quantities $N_1(t)$ and $N_2(t)$ at time t. Show that
   $$N_2(t) \;=\; N\frac{\tau_2}{(\tau_1-\tau_2)}\left(e^{-t/\tau_1}-e^{-t/\tau_2}\right),$$
   is the solution to the equations with the initial conditions, $N_1(0) = N, \quad N_2(0) = 0$. What is the corresponding expression for $N_1(t)$?
   If $\tau_1 \gg \tau_2$ what is the ratio of $N_(t)$ and $N_2(t)$ for time t where $\tau_2 \ll t \ll \tau_1$?.

4. Discuss the evidence leading to the prediction of the existence of the neutrino. Explain why the neutrino has spin-$\frac{1}{2}$ and why it was believed until very recently that the neutrino was massless.

5. Describe the mechanism of α-decay and explain why α-decay lifetimes can vary from a few milliseconds to billions of years depending on the Q-value of the decay and the nuclear charge of the daughter nucleus.

6. Describe the experiment by C.S. Wu on the β-decay of $^{60}_{27}$Co and explain how this leads to the conclusion that parity is violated in β-decay.

7. Explain how carbon dating works.