PHYS3002 - NUCLEI AND PARTICLES

Problem Sheet 1 - Due Feb. 9, 2015

1. α-particles with kinetic energy 10 MeV are incident on a gold ($^{197}_{79}$Au) foil of thickness 0.1 mm at a rate of 10^8 particles per second. A detector of area $10^{-2}\,m^2$ is placed at an angle of 90^0 to the direction of the incident α-particles at a distance of 1 m from the foil.
   Starting with the formula for the differential cross-section with respect to solid angle for Rutherford scattering, calculate how many α-particles per second reach the detector. [5]
   [The density of gold is $1.93 \times 10^4\,kg\,m^{-3}$.]

2. If the nuclear radius of gold is 10 fm, above what threshold energy of incident α-particles would you expect to see deviations from the Rutherford scattering formula? For incident particles with energies just above that threshold would you expect to see these deviations at small or large scattering angles? State your reason. [3]

3. In the original Geiger and Marsden experiment the scattering angle of the observed α-particles varied form 5^0 to 150^0. What fraction of these particles had a scattering angle greater than 90^0? [3]
$$\left[\int \frac{\sin(\theta)}{\sin^4(\theta/2)}\,d\theta \;=\; \frac{-2}{\sin^2(\theta/2)}.\right]$$

4. Electrons of energy 100 MeV are scattered off a Sn (tin) nucleus (atomic number Z=50). Assuming the model in which the charge distribution is constant for $r < R$ and zero for $r > R$, with $R = 4$ fm, calculate the differential cross-section (with respect to solid angle) at a scattering angle of 30^0. [4]

5. Electrons of energy 500 MeV are scattered off a nucleus. The differential cross-section has its first minimum at a scattering angle of 30^0. Make an estimate of the nuclear radius. [3]

6. Use the Semi-empirical Mass Formula to calculate the binding energies of the nuclei
$$^{238}_{92}\mathrm{U},\quad ^{145}_{57}\mathrm{La}\ \text{ and }\ ^{90}_{35}\mathrm{Br}.$$
   Hence calculate the energy released in the (spontaneous fission) reaction:
$$^{238}_{92}\mathrm{U} \;\to\; ^{145}_{57}\mathrm{La} + {}^{90}_{35}\mathrm{Br} + 3n. \qquad [4]$$

7. Using the Semi-empirical Mass Formula show that for fixed odd atomic mass number, A, the most stable isobar has a neutron to proton ratio given by
$$\frac{\mathrm{N}}{\mathrm{Z}} \;=\; 1 + \frac{1}{2}\frac{a_C}{a_A}\mathrm{A}^{2/3}.$$
   Hence determine the atomic number of the most stable isobar with A=103. [3]

Qualitative Questions

1. What is meant by an impact parameter?

2. Given the relation between the impact parameter, b, and the scattering angle, θ,

$$b = \frac{D}{2\tan\left(\frac{\theta}{2}\right)}$$

   where D is the distance of closest approach (at zero impact parameter) for incident particles of a given electric charge and kinetic energy, derive an expression for the differential cross-section with respect to solid angle, for α-particles with kinetic energy T, scattering off a nucleus of charge Ze.

3. What is meant by a form-factor? Explain qualitatively why a diffraction pattern is observed when a nucleus is bombarded with high energy electrons and discuss how information about the charge distribution within the nucleus can be deduced for the diffraction pattern.
   What information can be deduced by substituting the incident electrons with incident high-energy neutrons?

4. Explain the origin of the five terms in the Semi-empirical Mass Formula and (as far as possible) why the terms have their given form. What can we deduce about the nature of the strong forces between nucleons from the fact that the first term is proportional to A?

5. What is meant by the electric quadrupole moment of a nucleus.? What can one say about the shape of the charge distribution within a nucleus that has a non-zero electric quadrupole moment.